About Li-Yorke chaos from a dimensional theoretical perspective

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Li-Yorke pairs

- Let (X, d) be a metric space and $T : X \mapsto X$ be a continues transformation.
- $(x, y) \in X^2$ is a Li-York pair if

$$\limsup_{n \longmapsto \infty} d(T^n x, T^n y) > 0 \text{ and } \liminf_{n \longmapsto \infty} d(T^n x, T^n y) = 0$$

- Let $\mathfrak{L}(X, T) \subseteq X^2$ be the set of Li-York pairs
- A system is Li-Yorke chaotic if $\mathfrak{L}(X, T)$ is uncountable.

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Some results

- Li-Yorke (1975): A continuous map f on the unit interval having three periodic point is Li-Yorke chaotic.
- Kuchta-Smital (1989): A continuous map f on the unit interval having one Li-Yorke pair is Li-Yorke chaotic.
- Blanchard, Kolyada, Glasner and Maass (2002): Positive topological entropy implies Li-Yorke chaos.
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Hyperbolic systems

- Let X be a compact manifold and T be smooth with an hyperbolic invariant set Λ.
- (Λ, T) is Li-Yorke chaotic since a positive Lyapunov exponent implies positive entropy.
- Are there "many" Li-Yorke pairs? This is a question in the dimension theory of dynamical systems.
- We say that Li-Yorke pairs have full dimension if

$$\dim_H \mathfrak{L}(\Lambda, T) = \dim_H \Lambda \times \Lambda$$

where $\dim_H$ denotes the Hausdorff dimension.

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Symbolic dynamics

- Let (Σ,σ) be the full shift on n symbols. This system is Li-York chaotic.
- For $s\in\Sigma$ and a sequence n_k of natural numbers let

$$\Sigma_{n_k}(s)=\{s_1\bar{s}_2[n_1\ \mathit{free}]s_v s_{v+1}\bar{s}_{v+2}[n_2\ \mathit{free}]\dots\}\subseteq\Sigma$$

- If $s\in\Sigma$ and $t\in\Sigma_{n_k}(s)$ the pair (s,t) is Li-York pair hence

$$\mathfrak{L}_{n_k}(\Sigma,\sigma)=\{(s,t)|s\in\Sigma,\ t\in\Sigma_{n_k}(s)\}\subseteq\mathfrak{L}(\Sigma,\sigma)$$

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Iterated function systems

- Let $(T_1, ..., T_n)$ be contractions on X. Than there is unique compact invariant attractor Λ with

$$\Lambda = \bigcup_{i=1}^{n} T_i \Lambda \qquad \text{[Hutchinson 1981]}$$

- The coding map $\pi : \Sigma \mapsto \Lambda$ is given by

$$\pi((s_k)) = \lim_{n \longrightarrow \infty} T_{s_n} \circ \cdots \circ T_{s_2} \circ T_{s_1}(X)$$

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Main theorem

- Let $(T_1, ..., T_n)$ be an IFS with contracting similitudes and self-similar attractor Λ fulfilling the open set condition

$$T_i(O) \subseteq O \qquad T_i(O) \cap T_j(O) = \emptyset.$$

- Let n_k be a sequence of natural numbers with

$$\lim_{n \longmapsto \infty} \frac{\sum_{k=1}^{N} n_k}{N^2} = \infty$$

- Than for all $s \in \Sigma$ we have

$$\dim_H \pi(\Sigma_{n_k}(s)) = \dim_H \Lambda \text{ and } \dim_H \pi(\mathfrak{L}_{n_k}(\Sigma, \sigma)) = \dim_H \Lambda^2$$

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Consequences

- If (Λ, T) is homöomorphic conjugated to the shift (Σ, σ) and Λ is a self-similar invariant set than Li-Yorke pairs have full dimension.
- If (Λ, T) is homöomorphic conjugated to a two-sided full shift and the coding map is a product with two self-similar images, $\Lambda = \Lambda_1 \times \Lambda_2$, than Li-Yorke pairs have full dimension.
- For classical toy models of chaotic dynamics Li-Yorke pairs have full dimension, i.e. the repeller of the tend map, linear horseshoes and solenoids, the attractor of (generalized) Baker's transformations.

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- Generalize the result from linear to conform systems using thermodynamic formalism.

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Thanks for Your Attention!